Laboratorios de Maple
Geometría Diferencial MATE2410
Lab3 Representación de Weierstrass - Enneper II

Prof. José Ricardo ARTEAGA BEJARANO

1. Funciones holomorfas

1.1. Función holomorfa

```
> restart: with(plots):with(liesymm):with(linalg):
```

```
Warning, the name changecoords has been redefined

Warning, the protected name close has been redefined and unprotected

Warning, the protected names norm and trace have been redefined and
unprotected
```

```
> assume(u,real);
> assume(v,real);
> Phi:=z^(3);
> F(z):=subs(z=u+I*v,Phi);
```

$$\Phi := z^3$$

$$\mathrm{F}(z) := (u\tilde{} + I\, v\tilde{})^3$$

```
> phi(u,v):=evalc(Re(F(z)));
> psi(u,v):=evalc(Im(F(z)));
```

$$\phi(u\tilde{},\, v\tilde{}) := u\tilde{}^3 - 3\, u\tilde{}\, v\tilde{}^2$$

$$\psi(u\tilde{},\, v\tilde{}) := 3\, u\tilde{}^2\, v\tilde{} - v\tilde{}^3$$

```
> laplacian(phi(u,v), [u,v]);
> laplacian(psi(u,v), [u,v]);
> diff(phi(u,v),u)-diff(psi(u,v),v);
> diff(phi(u,v),v)+diff(psi(u,v),u);
```

$$0$$
$$0$$
$$0$$
$$0$$

1.2. Superfcie asociada

```
> X:=[u,v,phi(u,v),psi(u,v)];
> Xu:=diff(X,u);
> Xv:=diff(X,v);
> E:=simplify(dotprod(Xu,Xu));
> F:=simplify(dotprod(Xu,Xv));
> G:=simplify(dotprod(Xv,Xv));
> Xuu:=diff(Xu,u);
> Xuv:=diff(Xu,v);
> Xvv:=diff(Xv,v);
```

$$X := [u\tilde{},\, v\tilde{},\, u\tilde{}^3 - 3\, u\tilde{}\, v\tilde{}^2,\, 3\, u\tilde{}^2\, v\tilde{} - v\tilde{}^3]$$

$$Xu := [1,\, 0,\, 3\, u\tilde{}^2 - 3\, v\tilde{}^2,\, 6\, u\tilde{}\, v\tilde{}]$$

$$Xv := [0,\, 1,\, -6\, u\tilde{}\, v\tilde{},\, 3\, u\tilde{}^2 - 3\, v\tilde{}^2]$$

$$E := 1 + 9\, u\tilde{}^4 + 18\, u\tilde{}^2\, v\tilde{}^2 + 9\, v\tilde{}^4$$

$$F := 0$$

$$G := 1 + 9\, u\tilde{}^4 + 18\, u\tilde{}^2\, v\tilde{}^2 + 9\, v\tilde{}^4$$

$$Xuu := [0,\ 0,\ 6\,\tilde{u},\ 6\,\tilde{v}]$$

$$Xuv := [0,\ 0,\ -6\,\tilde{v},\ 6\,\tilde{u}]$$

$$Xvv := [0,\ 0,\ -6\,\tilde{u},\ -6\,\tilde{v}]$$

```
> H:=simplify((1+G)*(Xuu[3]+Xuu[4])+(1+E)*(Xvv[3]+Xvv[4])-2*F*(Xuv[3]+X
> uv[4]));
```

$$H := 0$$

2. Representación de Weierstrass - Enneper

```
> restart: with(plots): with(linalg):
```

```
Warning, the name changecoords has been redefined

Warning, the protected names norm and trace have been redefined and
unprotected
```

```
> F(tau):=1/(2*tau^2);
> X:=((int((1-tau^2)*F(tau),tau)));
> Y:=((int(I*(1+tau^2)*F(tau),tau)));
> Z:=((int(2*tau*F(tau),tau)));
```

$$\mathrm{F}(\tau) := \frac{1}{2}\,\frac{1}{\tau^2}$$

$$X := -\frac{1}{2}\,\tau - \frac{1}{2}\,\frac{1}{\tau}$$

$$Y := \frac{1}{2}\,I\,(\tau - \frac{1}{\tau})$$

$$Z := \ln(\tau)$$

```
> X:=subs(tau=exp(z),X);
> Y:=subs(tau=exp(z),Y);
> Z:=subs(tau=exp(z),Z);
```

$$X := -\frac{1}{2}\,e^{z} - \frac{1}{2}\,\frac{1}{e^{z}}$$

$$Y := \frac{1}{2}\,I\,(e^{z} - \frac{1}{e^{z}})$$

$$Z := \ln(e^{z})$$

```
> X:=subs(z=u+I*v,X);
> Y:=subs(z=u+I*v,Y);
> Z:=subs(z=u+I*v,Z);
```

$$X := -\frac{1}{2}\,e^{(u+I\,v)} - \frac{1}{2}\,\frac{1}{e^{(u+I\,v)}}$$

$$Y := \frac{1}{2}\,I\,(e^{(u+I\,v)} - \frac{1}{e^{(u+I\,v)}})$$

$$Z := \ln(e^{(u+I\,v)})$$

```
> X1Re:=simplify(factor(evalc(Re(X))),trig);
> X1Im:=simplify(factor(evalc(Im(X))),trig);
> X2Re:=simplify(factor(evalc(Re(Y))),trig);
> X2Im:=simplify(factor(evalc(Im(Y))),trig);
> X3Re:=simplify(factor(evalc(Re(Z))),trig);
> X3Im:=simplify(factor(evalc(Im(Z))),trig);
```

$$X1Re := -\frac{1}{2}\,\frac{\cos(v)\,((e^{u})^2+1)}{e^{u}}$$

$$X1Im := -\frac{1}{2}\,\frac{\sin(v)\,((e^{u})^2-1)}{e^{u}}$$

$$X2Re := -\frac{1}{2}\,\frac{\sin(v)\,((e^{u})^2+1)}{e^{u}}$$

$$X2Im := \frac{1}{2}\,\frac{\cos(v)\,((e^u)^2-1)}{e^u}$$

$$X3Re := u$$

$$X3Im := v$$

```
> RRR:=evalm([X1Re,X2Re,X3Re]);
> RRI:=[X1Im,X2Im,X3Im];
```

$$RRR := \left[-\frac{1}{2}\,\frac{\cos(v)\,((e^u)^2+1)}{e^u},\ -\frac{1}{2}\,\frac{\sin(v)\,((e^u)^2+1)}{e^u},\ u\right]$$

$$RRI := \left[-\frac{1}{2}\,\frac{\sin(v)\,((e^u)^2-1)}{e^u},\ \frac{1}{2}\,\frac{\cos(v)\,((e^u)^2-1)}{e^u},\ v\right]$$

```
> plot3d(RRR,u=-1.3..1.3,v=0..2*Pi,orientation=[134,86],numpoints=5000,
> style=PATCH,title=‘Superfcie mínima asociada a F(tau)‘);
```